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Solution by HOWARD C. FEEMSTER, A. B., York College, York, Neb.

Let the sides of the polygon a_1, a_2, ..., a_n turn severally around the n fixed points O_1, O_2, ..., O_n, describing n flat pencils, O_1, O_2, ..., O_n, while the $n-1$ vertices A_1, A_2, ..., A_{n-1} slide, respectively, along the $n-1$ fixed straight lines, v_1, v_2, ..., v_{n-1}, forming $n-1$ ranges, A_1, A_2, ..., A_{n-1}. Pencils O_1 and O_2 are perspective with range A_1; ranges A_1 and A_2 are perspective with pencil O_2; pencils O_2 and O_3 are perspective with range A_2, thus making the n pencils and $n-1$ ranges projective.

Now the projective pencils O_1 and O_n or any two of the non-consecutive pencils (using above order) will not, in general, be perspective, and every such intersection will thus describe a conic range as its locus. *Encyclopedia Britannica*, "Projective Geometry," Sec. 45.

347. Proposed by W. J. GREENSTREET, M. A. Marling School, Stroud, England.

ABC is a triangle, and D, E, F, are the mid points of the arcs of its nine-point circle cut off by BC, CA, AB, respectively. The inscribed circle touches these sides at X, Y, Z. Are the lines DX, EY, FZ concurrent? A purely geomatrical discussion required.

Solution by BENJ. F. FINKEL, Ph. D., Drury College, Springfield, Mo.

In my *Mathematical Solution Book*, page 461, it is shown that the perpendiculars to the chords cut off by the nine-point circle, at the midpoints intersect in a point S, which is the center of the nine-point circle. These perpendiculars produced from the chords to the subtended arcs, bisect the smaller arcs in D, E, and F. Since the in-circle is tangent to the chords or chords produced in X, Y, Z, perpendiculars erected at these points meet in a point, I, the center of the in-circle. Drawing the lines DX and EY, and producing them until they meet IS, or IS produced, in C and C', respectively, we have the triangles SDC and SDC'.

The triangles SDC and IXC are similar, as are also SDC' and IYC'. Hence, SD, (the radius of the nine-point circle): IX, (the radius of the in-circle) $=SC : IC$; and similarly, SE, the radius of the nine-point circle: IY, the radius of the in-circle, $=SC' : IC'$. Hence, $SD-IX : IX=SC-IC(=SI) : IC$, and $SE-IY : IY=SC'-IC'(=SI) : IC'$. Hence, $IC=IC'$ and, therefore, DX and EY intersect the line joining the nine-point circle and the in-circle in the common point C. In the same way, it may be shown that the line FZ intersects the same line in the point C. Hence, the three lines are concurrent.

CALCULUS.

275. Proposed by C. N. SCHMALL, 604 East 5th Street, New York City.

Explain fully why the circular measure of an angle is used in the calculus.

Solution by the PROPOSER.

There is one good reason for changing from the *gradual measure* to the *circular measure* in the calculus.

In finding the derivative of $\sin u$ we are required to evaluate

$$\underset{u \doteq 0}{\text{Lim.}} \left| \frac{\sin\frac{1}{2}u}{\frac{1}{2}u} \right|,$$

which becomes equal to unity.

Had we used degrees, we would have introduced the extraneous factor $\pi/360$. For, suppose u to be given in degrees, minutes or seconds; then we have

$$\frac{d}{dx}(\sin u^\circ) = \frac{\pi}{180} \cos u^\circ \frac{du}{dx}; \quad \frac{d}{dx}(\sin x') = \frac{\pi}{180.60} \cos x';$$

$$\frac{d}{dx}(\sin x'') = \frac{\pi}{180.60.60} \cos x''.$$

Taking a numerical example, let $x = 20^\circ$ and $\Delta x = .0001^\circ$.

From a table of natural sines, radius being 1, we obtain $\sin 20^\circ = .3420201 = u$, say; and, $\sin 20.0001^\circ = .34202174 = u + \Delta u$. $\therefore \Delta u = .00000164$.

Now, the length of a circular arc subtending an angle of $.0001^\circ$ is, (radius being 1) $.0000017453 = \Delta x$.

$$\therefore \frac{\Delta u}{\Delta x} = \frac{\text{increment of sine}}{\text{increment of arc}} = \frac{.00000164}{.0000017453} = .93967,$$

which is the value of $\cos 20^\circ$ correct to four places. The actual value, found by indefinitely decreasing the increment, is

$$\frac{du}{dx} = \cos 20^\circ = .9396926.$$

Remark by BENJAMIN F. FINKEL.

An angle, being a measurable magnitude, must be measured, or compared, by some arbitrarily chosen magnitude of its own kind. This arbitrarily chosen angular magnitude, we call the *unit angle.* In geometry, the unit angle is the angle formed by two straight lines perpendicular to each other. This unit is too large for trigonometric purposes, and so a smaller angle is chosen as a unit, namely, the 90th part of a right angle, called a degree. Any other arbitrarily chosen part would serve analysis as well, though the convenience or inconvenience in computation would be modified thereby. Now in elementary geometry it is proved that angles at the center are proportional to the intercepted arcs.

Let $s[L]$, $r[L]$, and θ be the arc, radius, and center angle, s and r being measured in the same *length-unit*, the angle unit being at present undetermined. Then $s[L]=kr[L]\theta$. Let us now *assume* $\theta=1$, *the unit angle, when* $s=r$. Hence, $k=1$, under this assumption. We thus have as our unit angle, $[\Theta]$, the angle at the center of a circle, which intercepts an arc equal in length to the radius of the circle. Hence, we have the general relation $s[L]=r[L]\,\theta\,[\Theta]$ and $\theta[\Theta]=s/r$. This unit angle, $[\Theta]$, is called a *radian*. In terms of the fundamental units of length, mass, and time, it is of zero dimensions, since $[\Theta]=\frac{s[L]}{r[L]}[M^\circ][T^\circ]=L^\circ M^\circ T^\circ$, s being equal to r. From this we see that the great advantage of this unit of angular measure over that of any other is that it avoids, as remarked by Mr. Schmall, the introduction of a *coefficient of variation* different from unity.

PROBLEMS FOR SOLUTION.

ALGEBRA.

329. Proposed by C. N. SCHMALL, 604 East 5th Street, New York City.

Between the quantities a and b there are inserted n arithmetical and n harmonical means, and a series of n terms is formed by dividing each arithmetical by the corresponding harmonical mean. Show that the sum of the series is, $n\left[1+\frac{n+2}{n+1}\cdot\frac{(a-b)^2}{6ab}\right]$.

330. Proposed by R. D. CARMICHAEL, Princeton, N. J.

An important function in the Theory of Numbers is one defined thus: $f(x)=1$ when $x>0$, $f(x)=0$ when $x=0$, $f(x)=-1$ when $x<0$. Two analytic expressions for $f(x)$ are the following:

$$f(x)=\lim_{n\doteq\infty} x^{1/(2n-1)},\ n=1,\ 2,\ \ldots;\quad f(x)=\lim_{n\doteq\infty}\frac{(x+1)^n-(x+1)^{-n}}{(x+1)^n+(x+1)^{-n}},\ x>-1.$$

It is required to find other non-trigonometric analytic expressions for this function. (There are several representations of $f(x)$ by means of trigonometric functions.)

GEOMETRY.

357. Proposed by E. R. HOYT, St. Louis, Mo.

A room is 30 feet long, 12 feet wide, and 12 feet high. At one end of the room, 3 feet from the floor, and midway from the sides, is a spider. At the other end, 9 feet from the floor, and midway from the sides, is a fly. Determine the shortest path by way of the floor, ends, sides, and ceiling, the spider can take to capture the fly.